Evaluating an Interesting Limit

Using $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$, calculate:

1. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{3n}$
2. $\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{5n}$
3. $\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{5n}$

19/7/25

1. $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{3n}$

$= \left(\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n}\right)^{3}$

$= e^{3}$

2. $\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^{5n}$

$= \lim_{n\to\infty} e^{\ln\left(1+\frac{2}{n}\right)^{5n}}$

$= e^{\lim_{n\to\infty} 5n\ln\left(1+\frac{2}{n}\right)}$

Let $n = \frac{2}{\Delta x}$.

$\Rightarrow \lim_{\Delta x\to 0} \frac{10}{\Delta x} \ln(1+\Delta x)$

$= \lim_{\Delta x\to 0} \frac{10\ln(1+\Delta x) - 10\ln 1}{\Delta x}$

$= 10 \frac{d}{dx} \ln 1$

$= 10$

$\therefore \lim_{n\to\infty} \left(1+\frac{2}{n}\right)^{5n} = e^{10}$

3. $\lim_{n\to\infty} \left(1+\frac{1}{2n}\right)^{5n}$

$= e^{\lim_{n\to\infty} 5n\ln\left(1+\frac{1}{2n}\right)}$

Let $n = \frac{1}{2\Delta x}$.

$\Rightarrow \lim_{\Delta x\to 0} \frac{5}{2} \cdot \frac{1}{\Delta x} \ln(1+\Delta x)$

$= \frac{5}{2} \lim_{\Delta x\to 0} \frac{\ln(1+\Delta x) - \ln 1}{\Delta x}$

$= \frac{5}{2} \frac{d}{dx} \ln 1$

$= \frac{5}{2}$

$\therefore \lim_{n\to\infty} \left(1+\frac{1}{2n}\right)^{5n}$

$= e^{\frac{5}{2}}$