

## Evaluating an Interesting Limit

Using  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$ , calculate:

1.  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{3n}$

2.  $\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^{5n}$

3.  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^{5n}$

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$$\begin{aligned}
 1. \quad & \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{3n} \\
 &= \left(\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n\right)^3 \\
 &= e^3
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^{5n} \\
 &= \lim_{n \rightarrow \infty} e^{\ln \left(1 + \frac{2}{n}\right)^{5n}} \\
 &= e^{\lim_{n \rightarrow \infty} 5n \ln \left(1 + \frac{2}{n}\right)}
 \end{aligned}$$

$$\text{Let } n = \frac{2}{\Delta x}$$

$$\begin{aligned}
 \Rightarrow & \lim_{\Delta x \rightarrow 0} \frac{10}{\Delta x} \ln(1 + \Delta x) \\
 &= \lim_{\Delta x \rightarrow 0} \frac{10 \ln(1 + \Delta x) - 10 \ln 1}{\Delta x} \\
 &= 10 \frac{d}{dx} \ln 1
 \end{aligned}$$

$$= 10$$

$$\therefore \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^{5n} = e^{10}$$

$$\begin{aligned}
 3. \quad & \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^{5n} \\
 &= e^{\lim_{n \rightarrow \infty} 5n \ln \left(1 + \frac{1}{2n}\right)}
 \end{aligned}$$

$$\text{Let } n = \frac{1}{2\Delta x}$$

$$\begin{aligned}
 \Rightarrow & \lim_{\Delta x \rightarrow 0} \frac{5}{2} \cdot \frac{1}{\Delta x} \ln(1 + \Delta x) \\
 &= \frac{5}{2} \lim_{\Delta x \rightarrow 0} \frac{\ln(1 + \Delta x) - \ln 1}{\Delta x} \\
 &= \frac{5}{2} \frac{d}{dx} \ln 1 \\
 &= \frac{5}{2}
 \end{aligned}$$

$$\begin{aligned}
 \therefore & \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^{5n} \\
 &= e^{\frac{5}{2}}
 \end{aligned}$$